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Three-Nucleon Low-Energy Constants from the Consistency of Interactions and Currents in Chiral Effective Field Theory

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The chiral low-energy constants c_D and c_E are constrained by means of accurate *ab initio* calculations of the $A=3$ binding energies and, for the first time, of the triton β decay. We demonstrate that these low-energy observables allow a robust determination of the two undetermined constants. The consistency of the interactions and currents in chiral effective field theory is key to this remarkable result. The two- plus three-nucleon interactions from chiral effective filed theory defined by properties of the $A=2$ system and the present determinantion of c_D and c_E are successful in predicting properties of the $A=3$, and 4 systems.

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The fundamental connection between the nuclear forces and the underlying theory of quantum chromodynamics (QCD) remains one of the greatest contemporary theoretical challenges, due to the non-perturbative character of QCD in the low-energy regime relevant to nuclear phenomena. However, the last two decades of theoretical developments provide us with a bridge to overcome this obstacle, in the form of chiral perturbation theory (χPT) [1]. The χPT Lagrangian, constructed by integrating out degrees of freedom of the order of $\Lambda_\chi \sim 1\,\mathrm{GeV}$ and higher (nucleons and pions are thus the only explicit degrees of freedom), is an effective Lagrangian of QCD at low energies. As such, it retains all assumed symmetry principles, particularly the approximate chiral symmetry of the underlying theory. Furthermore, it can be organized in terms of a perturbative expansion in positive powers of Q/Λ_χ where Q is the generic momentum in the nuclear process or the pion mass [1]. While chiral symmetry dictates the operator structure of each term of the effective Lagrangian, the coupling constants (not fixed by the symmetry) carry all the information on the integrated-out degrees of freedom. A theoretical evaluation of these coefficients, or low-energy constants (LECs), is equivalent to solving QCD at low-energy, and it is not yet feasible to obtain them from lattice calculations because of computational limitations. Alternatively, these undetermined constants can be constrained by low-energy experiments.

The strength of χPT is that the chiral expansion is used to derive both nuclear potentials and currents from the same Lagrangian. Therefore, the electroweak interactions in nuclei (which determine reaction rates in processes involving external probes) and the strong interaction dynamics (πN scattering, the NN interaction, the NNN interaction, etc.) are all based on the same theoretical grounds and rooted in the low-energy limits of QCD. In particular, χPT predicts, along with the NN interaction at the leading order (LO), a three-nucleon (NNN) interaction at the next-to-next-to-leading order or N^{2}LO [2], and even a four-nucleon force at the fourth order (N^{3}LO) [3]. At the same time, the LO nuclear current consists of (the standard) single-nucleon terms, while two-body currents, also known as meson-exchange currents (MEC), make their first appearance at N^{2}LO [6]. Up to the fourth order in the chiral expansion both the potential and the current are fully constrained by the parameters defining the NN interaction, with the exception of two "new" LECs, c_D and c_E. The latter, c_E, appears only in the potential as strength of the NNN contact term [see Fig. 1 (a)]. On the other hand, c_D manifests itself both in the contact term part of the NN-π-N three-nucleon interaction of Fig. 1 (a) and in the two-nucleon contact vertex with an external probe of the exchange currents [see Fig. 1 (b)].

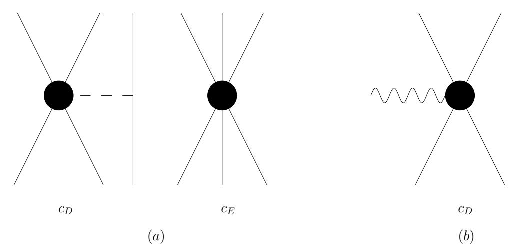

FIG. 1: Contact and one-pion exchange plus contact interaction (a), and contact MEC (b) terms of χPT at N^{2}LO.

The first determination of c_D and c_E was attempted using as constraints the ^{3}H binding energy (b.e.) and nd doublet scattering length, and adopting the full interaction up to N^{2}LO [4]. However, this proved to be difficult due to a correlation between these two observables, and the large experimental uncertainty on the scattering length. Later, the N^{3}LO NN potential was combined with the available NNN at N^{2}LO to study the ^{7}Li structure [5]. In this work, besides the ^{3}H b.e. the second constraint on the undetermined LECs was the energy of the ^{4}He ground state (g.s.). As a result of the correlation between these two observables, known as Tjon line, fitting the ^{3}H g.s. energy automatically results into a ^{4}He b.e. within few hundred keV off experiment. The subsequent fine-tuning of this b.e. is then very sensitive to the structure of the adopted NNN force. Hence small variations of the cutoff, different regularization schemes, missing terms of the interaction, etc., tend to produce large swings in the extracted values of c_D and c_E. A different approach was adopted in Ref. [7]. There, a preferred choice for the two LEC's was obtained by complementing the constraint on the $A=3$ b.e. with a sensitivity study on the radius of the α particle and on various properties of p-shell nuclei. The same interaction was then successfully utilized to predict the ^{4}He total photo-absorption cross section [8].

The results of these *ab initio* no-core shell model (NCSM) calculations, performed using the NN potential at N^{3}LO and the NNN interaction at N^{2}LO, represented a major step forward into underpinning the inter-nucleon interaction in the consistent approach provided by χPT. At the same time, a complemental determination is important, and it is desirable to perform it within the few-nucleon sector. In this respect, the relation (mandated by the chiral symmetry of QCD) between electroweak processes and NNN-force effects offers venues to achieve such a goal. This relation, manifested in χPT via the appearance of c_D in both the NN-π-N diagram of Fig. 1 (a) and the one in Fig. 1 (b), was first noticed four decades ago by Blin-Stoyle and Tint [9], and later expanded and clarified in the context of effective-field theory by Hanhart *et al.* [10], and Gårdestig and Phillips [11]. In particular, in Ref. [11] the authors suggest the triton beta-decay as one of the electroweak processes that could be used as input to fix the strength of the NNN force. It is the purpose of this *Letter* to undertake this task and show that by using the triton half life, as well as the $A=3$ b.e., one can constrain the two undetermined LECs within the three-nucleon sector, by means of fully converged *ab initio* calculations. We demonstrate that this determination is robust. The resulting chiral Lagrangian predicts, without any free parameters, various $A=3$, and 4 properties.

The triton is an unstable nucleus, which undergoes β-decay with a "comparative" half-life of $(fT_{1/2})_t = (1129.6 \pm 3)$ s, as reported by Akulov and Mamyrin [12]. Using the procedure discussed by Simpson [13], and later revisited by Schiavilla *et al.* [14], this quantity can be used to extract $\langle E_1^A\rangle = |\langle ^3\text{He}||E_1^A||^3\text{H}\rangle|$, the reduced matrix element of the $J=1$ electric multiple of the axial vector current, through

$$(fT_{1/2})_t = \frac{K/G_V^2}{(1-\delta_c) + 3\pi\,\frac{f_A}{f_V}\,\langle E_1^A\rangle^2}\,. \tag{1}$$

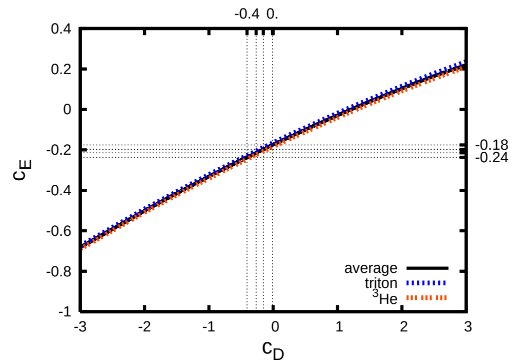

FIG. 2: (Color online.) c_D-c_E trajectories from fit to ^{3}H and ^{3}He experimental b.e. The dotted box binds the region for which $|1 - \langle E_1^A\rangle_{tho}/\langle E_1^A\rangle_{expt}|$ is within the experimental error-bars.

Here, $K = 2\pi^3 \ln 2/m_e^5$ (with m_e the electron mass), G_V is the weak interaction vector coupling constant (such that $K/G_V^2 = 6146.6 \pm 0.6$s [15]), $f_A/f_V = 1.00529$ [14] accounts for the small difference in the statistical rate function between vector and axial-vector transitions, and $\delta_c = 0.13\%$ [14] is a small correction to the reduced matrix element of the Fermi operator calculated between the $A=$ 3 wave functions (which is 1 for this specific case) due to isospin-breaking in the nuclear interaction. One can use these values to extract $\langle E_1^A\rangle|_{expt} = 0.6848 \pm 0.0011$.

The weak axial current adopted in this work is the Nöther current built from the axial symmetry of the chiral Lagrangian up to order N^{3}LO [6]. At LO this current consists of the standard single-nucleon part, which at low momentum transfer is proportional to the Gamov-Teller (GT) operator, $E_1^A|_{\rm LO} = i\, g_A (3\pi)^{-1/2}\sum_{i=1}^A \sigma_i\tau_i^+$, where σ_i, τ_i^+ are spin and isospin-raising operators of the ith nucleons, and $g_A = 1.2695 \pm 0.0029$ is the axial constant [16]. For this reason, the quantity $\sqrt{3\pi} g_A^{-1}\langle E_1^A\rangle|_{expt}$ is often referred to as "experimental" GT.

Corrections to the single-nucleon current appear at N^{2}LO in the form of MEC and relativistic terms. The MEC are formed by a one-(charged)-pion exchange, and a contact term. While the relativistic corrections are negligible for the triton half life, the MEC have a substantial influence on this β-decay rate. This is a reflection of the fact that E_1^A is a chirally unprotected operator [17]. Moreover, the strength of the MEC contact term, usually denoted by $\hat{d}_R$, is related to c_D through:

$$\hat{d}_R \equiv \frac{M_N}{\Lambda_\chi g_A} c_D + \frac{1}{3} M_N (c_3 + 2c_4) + \frac{1}{6}. \tag{2}$$

Here, M_N is the nucleon mass, and c_3, c_4 are LECs of the dimension-two πN Lagrangian, already part of the chiral NN potential at NLO. Therefore, one can use $\langle E_1^A\rangle|_{expt}$ as second constraint for the determination of c_D and c_E.

Following the c_D-c_E trajectory which reproduces the $A=3$ b.e. on average as discussed in Ref [7], here, we (i) calculate the ^{3}H and ^{3}He g.s. wave functions by solving the Schrödinger equation for three nucleons interacting via the χPT NN potential at N^{3}LO of Ref. [18] and the NNN interaction at N^{2}LO [2] in the local form of Ref. [19]; (ii) determine for which c_D values along the trajectory the calculated reduced matrix element of the E_1^A operator (at N^{3}LO) reproduces the experimental value.

The present calculations are performed in the framework of the NCSM approach [20]. This method looks for the the eigenvectors of the Hamiltonian in the form of expansions over a complete set of harmonic oscillator (HO) basis states up to a maximum excitation of $N_{\max}\hbar\Omega$ above the minimum energy configuration, where Ω is the HO parameter. The convergence to the exact results with increasing $N_{\max}$ is accelerated by the use of an effective interaction derived, in this case, from the adopted NN χPT potential at the two-body cluster level, to which we add the bare NNN force. Thanks to the large model-space size adopted ($N_{\max}=40$), $A=3$ b.e. and reduced matrix element of E_1^A are converged to less than 0.05%, and the same results can be obtained also trough fully bare (and variational) calculations [19]. Note that the same regulator $F_\Lambda(q^2)=\exp(-q^4/\Lambda^4)$ is used for both NNN terms of the interaction and MEC, a process resulting in a local chiral NNN force (for relevant parameters and definitions see Ref. [19]). The $A=3,4$ calculations of Ref. [19] were later confirmed by the results of Ref. [23], providing a benchmark for the local chiral NNN force. The MEC utilized in this work were validated against those of Park *et al.* [6]. Finally, we tested the implementation of the MEC within the NCSM approach by reproducing (within 0.1%) the AV18 results for $\langle E_A^1\rangle$ obtained with the effective-interaction hyperspherical harmonics technique.

As explained in Ref. [7] and shown in Fig. 2, there are infinite values of c_D and c_E that fit the triton b.e. These values sit on a one-dimensional curve in the c_D-c_E plane. Repeating this process for ^{3}He results in a slightly different curve. In the following, we will test the sensitivity of $\langle E_1^A\rangle$ to variations of c_D and c_E along the average of these two curves (solid line). The theory to experiment ratio for the E_1^A reduced matrix element in the range $-4\le c_D\le 10$ is presented in Fig. 3. The 2σ 1.08% tolerance band highlighted by the shaded area is mainly due to the uncertainies on $\langle E_1^A\rangle|_{expt.}$ and g_A. Besides the full calculation, which appears as a solid line, we report also the results of several tests, aimed to analyze the sensitivity of the triton half life to NNN force and/or MEC.

First we note the fundamental importance of the axial two-body currents in reaching agreement with experiment. By suppressing the MEC, in the whole investigated c_D-c_E range the calculations under-predict the half life by about 2%. The same almost constant behavior is found when adding to the single-nucleon current only the long-range one-pion-exchange term of the MEC, which corresponds to artificially setting $\hat{d}_R=0$. In this case, the

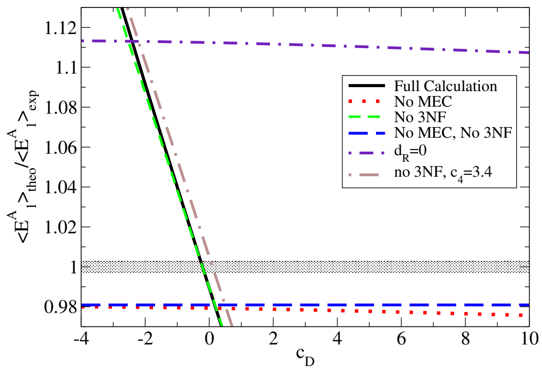

FIG. 3: (Color online.) Theory to experiment ratio for $\langle E_1^A\rangle$, using the N^{3}LO NN potential [18] with and without the local N^{2}LO NNN interaction [19], and the axial current with and without MEC for c_D, c_E along the averaged constraint of Fig. 2. The shaded area is the experimental uncertainty.

theoretical results over-predict the half-life by about 11%. Only when adding the contact part of the MEC, which is related to the short range weak correlations of axial character, can the half-life reach its experimental value. In particular, we find that the agreement within $\pm 0.54\%$ of experiment is obtained for $-0.3\le c_D\le -0.1$. The corresponding c_E values lie in the range $[-0.220,-0.189]$. These results are summarized in Fig. 2, where the dotted rectangle contains the segment of the c_D-c_E curve for which $\langle E_1^A\rangle|_{expt}$ is reproduced.

In a similar spirit, we now study the effect of the suppression of the NNN force. In this case, if we try to calibrate c_D to reproduce the measured half-life, we obtain a curve in close agreement with the results of the full calculation [11] (for completeness we show also the curve corresponding to the suppression of both MEC and NNN force). It is therefore clear that the half life of triton presents a very weak sensitivity to the NNN force, and hence to the strength of the spin-orbit interaction. Thanks to this feature, which is unique of the s-shell nuclei, we are confident that the determination of c_D and c_E obtained in this way is robust. Incidentally, the weak dependence of the half life of triton upon the NNN force can also explain the success of recent calculations done in a hybrid approach, coined EFT*, for s-shell nuclei [6].

As the values of the c_3 and c_4 LECs are somewhat uncertain (see, e.g., Ref. [5]), it is important to assess to which extent they would influence the determination of c_D from the triton half life. While very sensitive to the smallest change in c_3, the N^{3}LO fit of the NN data of Ref. [18] does not deteriorate dramatically for $3.4\,\mathrm{GeV}^{-1}\le c_4\le 5.4\,\mathrm{GeV}^{-1}$ [21]. Figure 2 shows calculations (without NNN force) carried out by artificially setting c_4 to $3.4\,\mathrm{GeV}^{-1}$ (πN value [22]) in the axial current, while the $A=3$ wave functions are still obtained from the N^{3}LO NN potential of Ref. [18] (where, in GeV^{-1}, $c_3=-3.2$ and $c_4=5.4$). We find that the use of the

TABLE I: Calculated ^{3}H, ^{3}He and ^{4}He g.s. energies (in MeV) and point-proton radii (in fm), obtained using the N^{3}LO NN potential [18] with and without the local N^{2}LO NNN interaction [19] with $c_D = -0.2$ and $c_E = -0.205$, compared to experiment.

	^{3}H		^{3}He		^{4}He	
	$E_{\mathrm{g.s.}}$	$\langle r_p^2 \rangle^{1/2}$	$E_{\mathrm{g.s.}}$	$\langle r_p^2 \rangle^{1/2}$	$E_{\mathrm{g.s.}}$	$\langle r_p^2 \rangle^{1/2}$
NN	−7.852(4)	1.651(5)	−7.124(4)	1.847(5)	−25.39(1)	1.515(2)
NN+NNN	−8.473(4)	1.605(5)	−7.727(4)	1.786(5)	−28.50(2)	1.461(2)
Expt.	−8.482	1.60	−7.718	1.77	−28.296	1.467(13) [24]

c_4 on the low side produces a shift ($\sim$0.3) towards more positive c_D values.

With this calibration of c_D and c_E, in principle, any other calculation is a prediction of χPT. In Table I we present a collection of $A = 3$ and 4 data, obtained with and without inclusion of the NNN force for $c_D = -0.2$ ($c_E = -0.205$), a choice roughly in the middle of the constrained interval. Besides triton and ^{3}He g.s. energies, which are by construction within few keV from experiment, the $NN + NNN$ results for the ^{4}He g.s. energy and point-proton radius are in good agreement with measurement. Note that the α particle g.s. energy varies from $-28.51(2)$ MeV to $-28.50(2)$ MeV for $c_D = -0.3$, and -0.1, respectively. The corresponding values for the point-proton radius, 1.460(2) fm and 1.462(2) fm, respectively, are both within the uncertainty of experiment. This results is not inconsistent with the study of mid-p-shell nuclei of Ref. [7], which showed preference for $c_D \sim -1$. For p-shell nuclei one should expect some re-normalization of the c_D value due to (neglected) higher-order NNN force terms, which are irrelevant for the calculation of the triton half life.

Summarizing, we have constrained the two undetermined N^{3}LO χPT parameters using properties of the three-nucleon system, namely the $A = 3$ b.e. and the half-life of triton. We find $-0.3 \leq c_D \leq -0.1$, and, correspondingly, $-0.220 \leq c_E \leq -0.189$. The weak sensitivity of the $\langle E_1^A \rangle$ matrix element with respect to the the NNN force makes it an excellent candidate for the determination of c_D. The next task is to take into account the N^{3}LO terms of the NNN interaction, which have not been included so far and will likely affect the determination of c_E. In addition, the half life of triton is somewhat sensitive to the πN LECs c_3 and c_4, the value of which is still under debate. Therefore, a sensitivity study in the $A = 2, 3$, and heavier systems for the c_4 and (correlated) c_D LECs is called for.

In conclusion, we have identified a clear path towards determining the NNN force that, once the NN interaction will be pinned down, will open the way to model-independent parameter-free predictions of QCD in the consistent approach provided by χPT.

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